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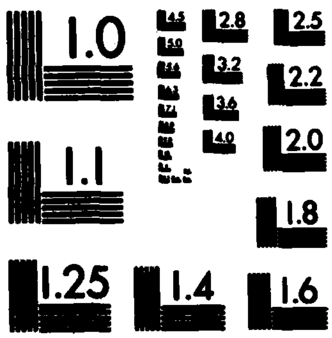

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Most of the research done under this contract can be described in terms of two guiding questions:

(i) Where is the boundary between integrable and nonintegrable problems?

(ii) What are the consequences of a problem's being on one side or the other of this boundary?

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"RESEARCH IN NONLINEAR MOTION"

FINAL REPORT

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June 30, 1984

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The development over the last twenty years of the theory of solitons and of completely integrable evolution equations has provided a new perspective from which to view certain problems in mathematics and in physics. An integrable problem has a great deal more structure than one ordinarily expects. This extra structure permits one to solve classes of integrable nonlinear problems in complete detail, a feat considered virtually impossible twenty years ago. The methods developed for integrable problems (inverse scattering, etc.) apply <u>only</u> to integrable problems, a fact that emphasizes the distinction between integrable and nonintegrable problems.

Aside from any question of method, the nature of the solution of an integrable problem also differs from that of a nonintegrable one. There is no loss of information as time advances (or recedes) in an integrable problem, so no uncertainly is introduced into the problem by the dynamics. In this sense, an integrable problem differs as much as possible from an ergodic one. Clearly it is important to be able to identify integrable problems, and to understand the consequences of a problem's being integrable or not. Most of the research done under this contract can be described in terms of two guiding questions:

(i) Where is the boundary between integrable and nonintegrable problems?

(ii) What are the consequences of a problem's being on one side or the other of this boundary?

The research under this contract resulted in seven publications, which are listed in the reference. These papers relate to the two guiding questions in the following way. (The subsequent discussion refers to those papers by their numbers in the list).

1. Integrability of Ordinary Differential Equations

Previous work (Ablowitz, Ramani and Segur, 1978, 1980) showed a deep connection between partial differential equations solvable by some inverse scattering transform and ordinary differential equations (ODE's) with the Painleve property (that the only movable singularities in the complex plane are poles). Although it was not stated explicitly in that work, this connection strongly suggests a related connection between integrable ODE's and those with the Painlevè property. In three papers (Refs. 1, 2, 3) it was demonstrated with nontrivial examples that the Painlevè property is effective in identifying integrable ODE's, i.e., if a system of ODE's has the Painlevè property, then it is integrable. The examples included both dissipative (Ref. 1) and Hamiltonian (Refs. 2, 3) systems. Integrability was established either by producing the extra integrals explicitly, or by demonstrating by means of numerical integration that the solutions lie on a lower dimensional manifold.

The objective of these papers was to demonstrate the effectiveness of the method, by finding new integrable cases in well-known problems. The deeper question is, Why should the Painlevè property imply integrability? This question remains open, and work on it continues.

2. Inverse Scattering in Higher Dimensions

The inverse scattering transform (IST) has been developed since 1967 to solve certain nonlinear problems that are integrable. For the most part, the problems are partial differential equations in one spatial and one temporal (1+1) dimensions. The basic mathematical theory in (1+1) dimension is now reasonably complete, although the discovery of new cases continues. In higher dimensions, such as (2+1) or (3+1) dimensions, very little is known.

The first clear work on IST in higher dimensions was done by Manakov (1981), who solved one version of the Kadomtsev-Petviashvili equation,

$$(u_t + 6uu_x + u_{xxx})_x = 3u_{yy}, \qquad \text{(KP1)}$$

on the plane $(-\infty < x, y < \infty)$. Manakov's results were formal, but their validity was established rigorously in Ref. 4 for initial data that are small enough in a certain morn.

An interesting aspect of (KP1) is that it admits "lumps", exact solutions that are spatially localized (in (2+1) dimensions, as solitons are not) and interact like solitons. Lumps are necessarily excluded from any solution obtained by Manakov's method, but this restriction (to small initial data) was removed in Ref. 4 and by Fokas and Ablowitz (1983). The final result is that for a wide class of initial data on $(-\infty < x, y < \infty)$ that decay as $x^2 + y^2 \to \infty$, the solution of (KP1) evolves into N lumps plus algebraically decaying radiation. This is identical with the qualitative picture in (1+1) dimensions, except that one-dimensional solitons are replaced by two dimensional lumps for KP1.

The number of higher dimensional problems that are known to be integrable is surprisingly small. Perhaps the main value of solving (KP1) by IST is that it provides one concrete example of the structure of an integrable problem in higher dimensions.

3. Periodic Waves in Shallow Water

The other version of the Kadomtsev-Petviashvili equation is

$$(u_t + 6uu_x + u_{xxx})_x + 3u_{yy} = 0. \qquad \text{(KP2)}$$

Both equations have physical significance, and both are integrable. KP2 describes the evolution of gravity-induced water waves of moderate amplitude as they propagate primarily in one direction in shallow water. Its solutions

live in (2+1) dimensions, and therefore it might reasonably describe waves that propagate on the two-dimensional water surface. In Ref. 6 is described a preliminary version of an explicit, analytical model of nonlinear, periodic, two-dimensional waves in shallow water. Preliminary comparisons of the waves predicted by this model with observed wave patterns in shallow water are very encouraging. A final version of this model should be completed shortly after the termination of this contract.

4. Integrability in Physical Models

Some integrable problems model physical phenomena, but the models usually are approximate, and sometimes they are only qualitatively suggestive. The physical systems are almost certainly not integrable. How can the knowledge obtained from a study of integrable systems best be used to interpret physical phenomena? Three papers (Refs. 5, 6, 7) dealt with aspects of this problem.

(a) Wobbling Kinks in Polyacetylene Molecules

Polyacetylene, $(CH)_x$, is a polymer (i.e., a plastic) made up of very long molecules, and whose electrical conductivity under light doping is comparable to that of a metal. There is widespread and intense interest in the material, and nearly every issue of Physical Review Letters in the last year has had at least one article related to polyacetylene.

The best current theory to explain the anomalous conductivity of polyacetylene is that charge-carrying solitary waves are easily excited on these long molecules, and that the abnormally large currents observed macroscopically are made up of many solitary waves, running along many molecules. (These solitary waves are called "solitons" in the jargon of solid-state physics, despite the objections of linguistic prints.) One model represents the waves running along a molecule by solutions of the so-called ϕ^4-equation,

$$\phi_{tt} - \phi_{xx} = \phi - \phi^3.$$

The work in Ref. 5 establishes rigorously a conjecture of Rice (1979) on the internal structure of these "solitons". Some of the interest from the perspective described in this report is that the ϕ^4- model is not integrable. The work in Ref. 5 shows how certain concepts from the integrable theory can be carried over to a nonintegrable problem.

(b) Kinetic Theory of Triads

In many physical systems, the simplest (weakly) nonlinear coupling of (nearly) linear Fourier modes is through "resonant triads." These problems become linear at lowest order in a small-amplitude expansion, with solutions of the form

$$\phi = \sum a_n \exp (i\vec{k}_n \cdot \vec{x} - i \omega_n t),$$

where ω_n is related to $\vec{k}_n$ through the dispersion relation of the linearized problem,

$$\omega = \omega (\vec{k}).$$

Resonant interactions occur among triads of those linear modes that satisfy resonance conditions of the form

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0, \qquad \omega_1 + \omega_2 + \omega_3 = 0$$

Resonant triads are particularly intriguing from the standpoint of integrable vs. nonintegrable problems, because triads have been the basis both of completely integrable models (Kaup, 1980) and of irreversible, dissipative models (Hasselmann, 1966). One possible resolution of this paradox was developed in Ref. 7, in which an irreversible kinetic theory for wave packets is built up from a large collection of uncorrelated triad interactions, each of which is completely reversible. In this problem the individual wave packets, their triad interactions and the dilute gas of packets all live in (3+1) dimensions, so there is no artificial restriction on the dimensionality

of the problem.

This completes the summary of the research accomplished under this contract. It should also be noted that this list of accomplishments does not coincide with the list of problems posed three years ago. The discrepancies are because: (a) someone else solved the problem before I got to it; (b) someone else is working on the problem with a method that I consider better than mine; (c) some other problem arose during duration of the contract that appeared more interesting or more important than the one originally posed.

Here is a list of the problems posed three years ago, and their current status.

(i) Prove rigorously the connection formulae for the bounded real solutions of Painlevé's second equation (P_{II}), as given by Segur & Ablowitz (1981).

Martin Kruskal has developed a method within the last two years that seems to be capable of both finding and proving connection formulae for all of the Painlevé transcendents. Exploitation of this method will be the PhD thesis of Nalini Joshi, a student of Kruskal's at Princeton.

(ii) Find the location of the right most singularity in a solution of P_{II} that is bounded as $Z \to +\infty$, in terms of its asymptotic behavior as $Z \to +\infty$.

The Kruskal-Joshi method seems to be the best approach to this problem as well.

(iii) Study the relation between the methods proposed by Ablowitz, Ramani & Segur (1978, 1980) and by Zakharov & Shulman (1980) to identify partial differential equations that are completely integrable.

The method of Ablowitz, Ramani & Segur was generalized and improved by Weiss, Tabor & Carnevale (1982). Both methods depend on the Painlevé property as a test for integrability, but it is still not clear why the Painlevé property should be effective. The method of Zakharov and Shulman (1980)

itself is a mystery to me, let alone its relation to any other method. It can honestly be said that no successful research on this particular topic was done under this contract.

(iv) Find the asymptotic ($t \to +\infty$) behavior of the solution of the Toda lattice in terms of its initial data.

This problem was solved by McCoy, Perk & Shrock (1983).

(v) Study the relation between nonlinear ordinary differential equations that exhibit chaotic behavior and those of P-type.

Refs. 1, 2 and 3 addressed this topic, as discussed above.

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